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Solving Optimal Control Problems by Numerical Simulations: A Literature Review

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Resumo: Este artigo apresenta uma breve revisão da bibliografia relacionada à aplicação de simulações numéricas na solução de problemas de controle ótimos (OCP). Revisamos conceitos básicos, tais como os teoremas de existência e as condições suficientes para a otimização, no que diz respeito à otimização aplicada à engenharia de controle, bem como as principais regras relacionadas às simulações numéricas. O trabalho é baseado em pesquisas da literatura relevante publicada sobre o tema, a partir dos livros e artigos clássicos, e abrange as obras mais citadas na atualidade. Os resultados desta revisão sugerem que há uma falta de métodos numéricos para resolver OCP. Estes métodos parecem não fazer uso dos principais conceitos teóricos desenvolvidos nos últimos anos e, igualmente, parecem não explorar o poder computacional moderno. Este trabalho contribui para o conhecimento principal sobre o assunto, sugerindo que a melhor exploração do poder das simulações numéricas para resolver OCP, com o uso de métodos mais adequados, pode resultar em soluções de OCP muito relevantes, que são de grande interesse em sistemas de engenharia e até mesmo biológicos.

Palavras-Chave: Otimização Dinâmica; Controle Ótimo; Dengue; Modelica; Aedes Aegypti.

Abstract: This paper presents a short review of bibliography related to the application of numerical simulations to solve optimal control problems (OCP). We revised basic concepts, such as the existence theorems and the sufficient conditions for optimality, regarding optimization applied to control engineering, as well as the main rules related to numerical simulations. The work is based on research of the main literature published on the topic, starting from the classic books and articles, and covering the most recent and cited works. The results of this review suggest that there is a lack of the numerical methods to solve OCP. These methods seem do not make use of the main theoretical concepts developed in the recent years and, equally, seem do not explore the modern computational power. This work contributes to the main knowledge about the subject, by suggesting that a better exploitation of numerical simulations capabilities to solve OCP, with the use of more suitable methods, may result in solutions of very important OCP that are of high interest in engineering and even in biological systems.

Keywords: Dynamic Optimization; Optimal Control; Dengue; Modelica; Aedes Aegypti.

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1 INTRODUCTION

The context of this work stands on the intersection of the knowledge related to optimal control and numerical simulations.

The optimal control theory, (KIRK, 1970), (BRYSON, 1975), (ATHANS, 2007), has been developed from initially simple problems that clearly conform to classical theory requirements (CESARI, 1983), as the Existence Theorems about convexity and linearity, to ensure that optimal solution could be found. In order to support this approach, several methods were introduced and improved over the years, for analytical and numerical solutions, most of them hard tied to the definition of the problem.

This methods do not cover a wide part of control problems that could not be addressed by the theory, despite these OCP are of economic and social interest, to which there was no guide to easily reach practical solutions (ESTEVA; YANG, 2005), (THOMÉ, 2007). This lack of theory motivated a recent theory development which was not followed by numerical methods and techniques, and did not make exhaustive use of modern computational resources.

This short literature review aims to help the research of a method to use numerical simulations to solve OCP that do not conform to classical control theory.

The result of this work is the identification of this gap in the theory as an opportunity to find a solution to ill-fitted OCP. In our work we found the Aedes Aegypti control problem, the OCP related to the transmitter of Dengue and Zica virus, as an example where the theory and practice may move forward.

This OCP was chosen for its ill-fitted performance index, nonlinearity and mathematical complexity, as well as for the world concern about a global epidemic risk of huge social and economic impact.

Section 2 presents the literature review, from classical references to recent most cited articles. Section 3 outlines the methodology used in this work. The next section shows the results, covering the algorithm applied to the cited OCP. Section 5 discusses the consequences of the work for the control researchers and for everyone interested in solving ill-fitted OCP.

2 THEORETICAL FOUNDATIONS

The control theory is presented to engineering students by classical references (OGATA, 1993), (OGATA, 1995), (FRANKLIN; POWELL, 1980), (FRANKLIN et al., 1991)i . At this stage, basic concepts, definitions and classifications are established by the conventional control theory. This theoretical basis is founded over time and frequency domain and, for linear and time-invariant systems, over the transference function concept.

The main methods of the conventional control theory are the control solution by Root Locus, Project and Compensation and Frequency Analysis.

With the raising of modern control theory, there was a change from the approach of single variable to multiple variables with the concept of state space. This new theory uses concepts and definitions of controllability, observability and stability. Classical principles are presented, for example the Lyapunov's stability conditions. The model predictive control emerged in this context, as well as the adaptive control and the optimal control theory.

The optimal control theory aims to minimize a performance index, most of the time, related to an economic cost function.

The optimal control is the nucleus of our research and there is a large number of excellent bibliographic sources, (KIRK, 1970), (BRYSON, 1975), (ATHANS, 2007), that show several methods and techniques to solve specific OCP. These methods and techniques are, mostly, supported by numerical simulations to solve Ordinary Differential Equations, or more specifically, Riccati Equation and Hamilton-Jacobi-Bellman Equation. Some optimality principles were developed, such as Pontryagin Minimum Principle and Bellman Optimality Principle, used by the Dynamic Programming Technique (BERTSEKAS, 2005).

Most of these methods and techniques are implemented by numerical simulations and stands over the Existence Theorems (CESARI, 1983) to guarantee the optimal solution. However, despite the large number of methods and techniques to solve OCP, there is a huge number of OCP that do not fit classical control theory of linearity and convexity, as does the example. This lack of theory motivated the recent theoretical development further to consider nonconvexity and nonlinearity.

The article (RAFIKOV; BALTHAZAR, 2008) shows a refined innovation with an analytical solution for systems that are partially nonlinear. The analytical solution presented

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i We show here few classical references, since there is a large number of excellent bibliography.

handles, regards and comprises some of the system characteristics in the performance index.
Other important recent references, (TODOROV, 2005) to (CIMEN; BANKS, 2004), address the development of other areas of optimal control theory.

Even with recent approaches, (ZASLAVSKI, 2013), (CLARKE, 2013), (RAFIKOV; BALTHAZAR, 2008), that raised new possibilities, the solutions, analytical or numerical, of the OCP solved by the classical approach, are linked to the constraints that are imposed to the system and to the performance index as well.

These constraints are natural. However, the interesting point is that the present stage of optimal control theory shows a huge evolution in the existence theorems, where the not compliance to the classical conditions (CESARI, 1974), (ANCONA et al., 2008), (CELLINA; COLOMBO, 1990), (MARICONDA, 1992), (ZASLAVSKI, 2007), (CLARKE, 1983), (CLARKE, 2011), such as convexity and monotonicity, under some conditions, does not compromise the existence of the optimal solution, even if it is hard to find it.

With the surface of new concepts and approaches defined by recent theorems of existence, the theoretical evolution was not followed by a practical development in obtaining solutions. As the possibilities for objective functions were increased, they need not to be so tied to the constraints of classical theorems.

Other point to consider is related to the advances in the optimization itself. The optimization as mathematics field has been the object of recent theoretic and practical improvements. From the well-known Karush, Kuhn and Tucker Conditions (KKT) (BAZARAA; SHETTY, 1979), (LUENBERGER, 1984), several algorithms to solve an optimal problem (OP)ii have been proposed.

Recently, the dynamic optimization by collocation methods was presented as a good method to solve OCP, standing over the concept of transformation of a dynamic OCP in a huge static nonlinear program (NLP).

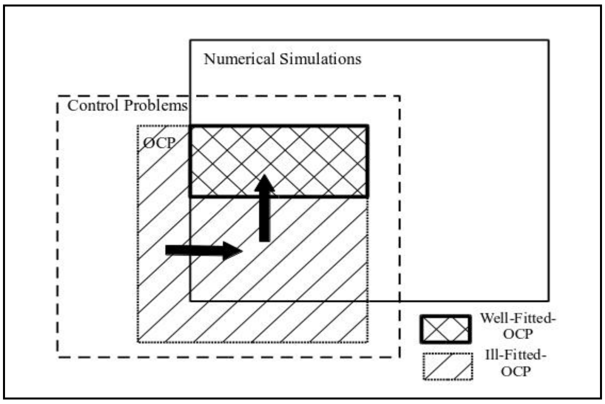
In this area, fundamental concepts related to the numerical methods to solve NLP must be known in order to make good use of computational resources. Modern tools like IPOPT (WÄCHTER; BIEGLER, 2006) and HSL (SCIENCE AND TECHNOLOGY FACILITIES COUNCIL, 2015) linear solvers make use of classical algorithms (BAZARAA; JARVIS; SHERALI, 2009) to numerically reach the optimal solution. The interior-point and barrier methods, concepts and definitions, as Lagrange multipliers, must be in the mind of the researcher just in time to choose the parameters that will be used by numerical solvers. With a bad handling of these parameters, the researcher will get no convergence to the optimization algorithm.

Another concern is the coding of the OCP model and its translation to the simulation tool. This process must be consistent. Furthermore, the results of simulations must be analyzed taking into account the tolerances and discretization parameters used.

Our work addresses this gap: it uses the potential of meta-programming, versatility and flexibility of the Modelica language models and computer automation in numerical simulations for instrumenting the study of these newly developed concepts in recent years. For example, nonconvexity may not mean impediment to obtain the optimal solution.

The (Fig. 1) graphically describes our understanding of the contribution of this work iii.

Figure 1 - Graphical description of the research.



Source: the authors.

3 METHODOLOGY

This bibliographic review regards some of the main classical literature on control engineering references (OGATA, 1993), (OGATA, 1995), (FRANKLIN; POWELL, 1980), (FRANKLIN et al., 1991). Since, in this work, it is not possible to explore deeply the explanation of each concept and definition shown in the discussed references,

ii We shall point out the difference of OCP and OP, basically, by the time variable. We do not have time dependence in OP, so OP is essentially a static problem. On other hand, OCP has a time dependence and represents a dynamic problem.

iii We must clarify that there are several ways to perform the movement suggested by the arrows. Our approach is to preserve in the OCP, as far as possible, the nature of the real problem and use parameters to adjust the performance index domain.

we cited the most important that were fundamental for the research.

After the initial bibliographic research, we made Internet searches on sites Scopus and ISI Web of Knowledge iv by the recent most cited articles in the field. The first search was filtered by the keywords ``Control Problems" and ``Numerical Simulations". The second search used ``Optimal Control" and ``Numerical Simulations". In both cases, we filtered the results to the fields: ``Computer Science", ``Mathematics" and ``Engineering".

The results are presented in the (Tab. 1).

Table 1 - Web Search Results

Search	Scopus	ISI Web of Science
1	4034	125
2	4034	536

All these articles were studied in their abstracts and classified by topics and kind of contribution, if theory development or application. We summarized this analysis for the most cited articles in (Tab. 2).

Other important point to regard is related to the main journals in the field. We presented in (Tab. 3), the most important magazines covering the topics in (Fig. 1).

Table 3 - Web Search Results

#	Journal	JIF	Country
1	Intern. Journal of Robust and Nonlinear Control	3.176	USA
2	Automatica	3.020	USA
3	IEEE Transactions on Automatic Control	2.770	USA
4	Journal of Process Control	2.653	UK
5	Annual Reviews in Control	2.518	USA
6	IEEE Transactions on Control Systems Technology	2.474	USA
7	Systems and Control Letters	2.059	NET
8	IET Control Theory & Applications	2.048	UK
9	Medical Engineering and Physics	1.825	UK
10	International Journal of Control	1.654	UK
11	Biomedical Signal Processing and Control	1.419	UK
12	Computer Methods and Programs in Biomedicine	1.897	NET
13	Computers in Biology and Medicine	1.240	USA
14	Journal of Vibration and Control	4.355	UK
15	Commun. in Nonlinear Science and Numer. Simul.	2.866	NET
16	Applied Mathematical Modelling	2.251	USA
17	Journal of Global Optimization	1.287	NET
18	Journal of Control, Autom. and Electrical Systems	*	BRA
19	Revista de Controle e Automacão	*	BRA

* Brazilian Journals with no JIF

Choosing the first journal in (Tab. 3), we can summarize the relationship with other journals by (Fig. 2) and (Fig. 3).

Figure 2 - Main journals that cited IJRNC - Total 3232 cites. Source: web access on May, 21st, 2016.

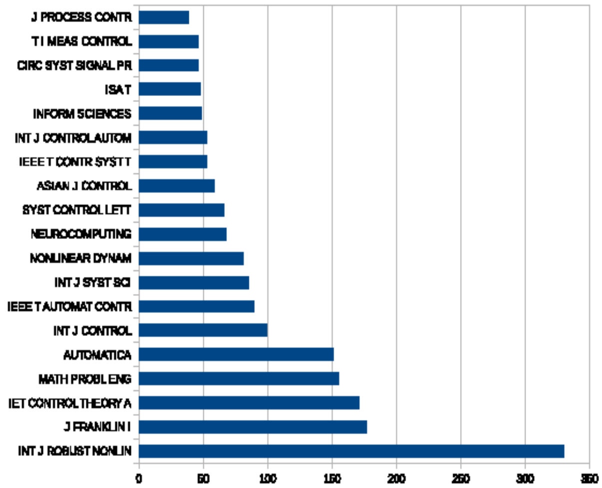
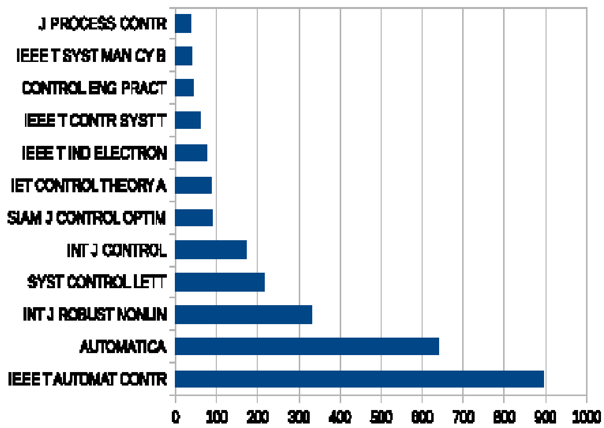


Figure 3 - Main journals cited by IJRNC - Total 5818 cites. Source: web access on May, 21st, 2016.



iv Access on May, 6th, 2016.

Table 2 - Most Cited Articles

#	Article	Cites	Remarks
1	(TODOROV, 2005)	127	theory development stochastic optimal control coordin.-descend algorithm
2	(RAFIKOV; BALTHAZAR, 2008)	121	theory development nonlinear systems chaos synchronization linear feedback controller
3	(LUO; CHU; LING, 2005)	106	application tracking control problem
4	(CHESI et al., 2003)	94	theory development
5	(JOO; SHIEH; CHEN, 1999)	91	theory development control of chaotic systems fuzzy controller
6	(SHAO; XIE; WANG, 2007)	78	application
7	(LIN; ZHANG; BRANDT, 1999)	77	application
8	(BANI-HANI; GHABOUSSI, 1998)	69	theory development neural networks nonlinear control
9	(BEWLEY; TEMAN; ZIANE, 2000)	66	application linear / nonlinear problems
10	(LI; FENG; LIAO, 2007)	60	theory development
11	(HOUSKA; FERREAU; DIEHL, 2011)	59	application NMPC
12	(ILZHOEFER; HOUSKA; DIEHL, 2007)	59	application NMPC
13	(CIMEN; BANKS, 2004)	57	theory development ASRE
14	(LIU; XIE; WANG, 2012)	56	application MASC
15	(CHOI et al., 2004)	53	application
16	(ZHOU et al., 1995)	47	application
17	(ERKUS; ABE; FUJINO, 2002)	45	application
18	(DING; JIA; WANG, 2004)	40	application
19	(BONNARD; CHYBA; SUGNY, 2009)	39	application
20	(LENCI; REGA, 2003)	35	theory development

NMPC: Nonlinear Model Predictive Control

ASRE: Approximating Sequence of Riccatti Equations

MASC: Multi-Agent Systems Contro

4 RESULTS

The analysis helped us to identify a gap in the optimal control theory that cannot cover OCP that is not well-fitted to classical existence theorems, despite the ill-fitted OCP may be of great economic and social interest.

As example, we may show the OCP related to Aedes Aegypti control as presented by (ESTEVA; YANG, 2005), (THOMÉ, 2007).

We think that the OCP presented in these works does not easily conform to classical theory: The problem is nonlinear and the performance index is a nonconvex function, reasons why it is a difficult problem to get the optimal solution.

Therefore, this OCP is an important problem that demands solutions that cannot be easily provided only by classical theory. We think that modern numerical simulations techniques , (ANDERSSON et al., 2011), (MAGNUSSON; ÅKESSON, 2012), can be used to handle this ill-fitted OCP, and we must research how to efficiently use these techniques in a generalized method.

5 DISCUSSION

The pointed gap in the literature is an important research field that may be explored by researchers in the area.

Our research suggests, see (Tab. 2), that most cited articles cover mainly theory development. Naturally the application is also very important, however, theory development is the basis for all the applications and, for this reason, articles on theory usually are widely cited.

We may see improvements in some techniques to solve specific problems. For example, in neural networks applications to optimal control, as well in nonlinear systems, robust control, adaptive control, partial differential equations and polynomial chaos.

All these fields of knowledge are explored by recent articles, most of them are based on the use of well-known MATLAB®´s routines and toolboxes.

We think that other numerical simulations techniques and tools, (ANDERSSON et al., 2011), (MAGNUSSON; ÅKESSON, 2012), (ÅKESSON et al., 2010), (WÄCHTER; BIEGLER, 2006), (SCIENCE AND TECHNOLOGY FACILITIES COUNCIL, 2015), should be explored by the scientific community to go further in theory development and applications.

Thinking specifically about the OCP cited in the Section 3, the first idea is to change the performance index to adapt to classical theory and apply the recent methods (RAFIKOV; BALTHAZAR, 2008) to solve nonlinear OCP. This was done by (RAFIKOV; RAFIKOVA; YANG, 2015). However, we suggest that an alternative option could be to preserve the performance index as far as possible and use numerical simulations to solve the problem. This was done in (RESENDE, 2015)

6 CONCLUSIONS

This work showed the results obtained by a short literature review in numerical simulations applied to optimal control.

We found a gap in the existent literature that does not cover OCP with ill-fitted performance indexes. The results suggest a new perspective for handling ill-fitted performance index by numerical simulation and analysis, as an interesting field to be explored by control researchers. Furthermore, the possibility to solve ill-fitted OCP, as shown for the OCP related to Aedes Aegypti control, may have positive social and economic impact.

Future works may consider investigating other specific ill-fitted OCP that are of interest to the scientific community, for example, the OCP related to the SIRC epidemic model (IACOVIELLO; STASIO, 2013).

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